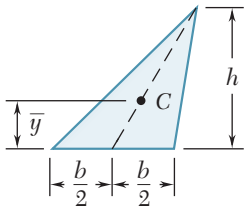
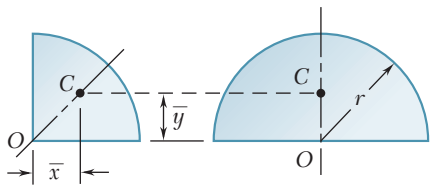
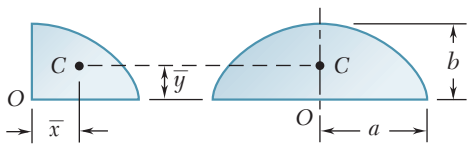
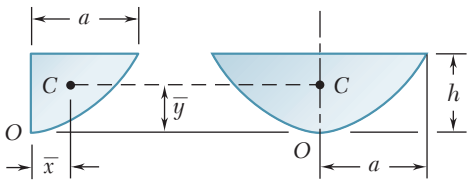
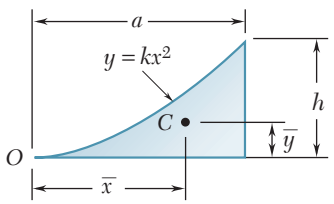
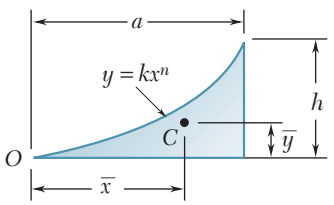
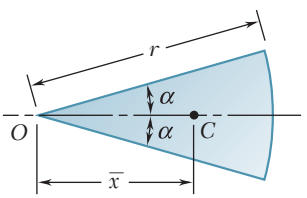
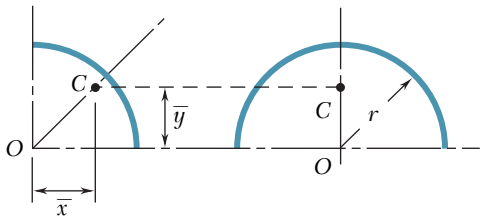
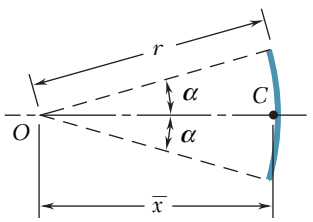
Centroides de áreas comunes

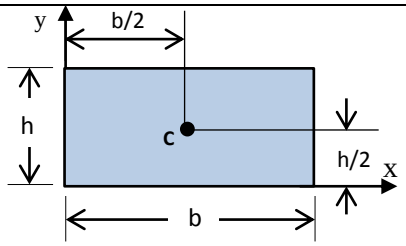
Forma		$\bar{x}$	$\bar{y}$	Área
Área triangular			$\frac{h}{3}$	$\frac{bh}{2}$
Un cuarto de área circular		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Área semicircular		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Un cuarto de área elíptica		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Área semielíptica		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Área semiparabólica		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Área parabólica		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Enjuta parabólica		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Enjuta general		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Sector circular		$\frac{2r \operatorname{sen} \alpha}{3\alpha}$	0	αr^2

Centroides de formas comunes de líneas

Forma		$\bar{x}$	$\bar{y}$	Longitud
Un cuarto de arco circular		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Arco semicircular		0	$\frac{2r}{\pi}$	πr
Arco de círculo		$\frac{r \operatorname{sen} \alpha}{\alpha}$	0	$2\alpha r$

Momentos de inercia de áreas – Mecánica racional I

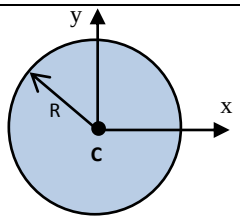
Rectángulo



$$\bar{I}_x = \frac{bh^3}{12} \qquad \bar{I}_y = \frac{b^3h}{12} \qquad \bar{I}_{xy} = 0$$

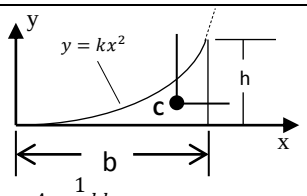
$$I_x = \frac{bh^3}{3} \qquad I_y = \frac{b^3h}{3} \qquad I_{xy} = \frac{b^2h^2}{4}$$

Círculo



$$I_x = I_y = \frac{\pi R^4}{4} \qquad I_{xy} = 0$$

Media Parabólica complementaria



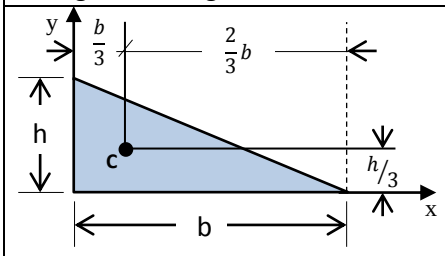
$$\bar{x} = \frac{3b}{4} \qquad \bar{y} = \frac{3h}{10} \qquad A = \frac{1}{3}bh$$

$$\bar{I}_x = \frac{37bh^3}{2100} \qquad I_x = \frac{bh^3}{21}$$

$$\bar{I}_y = \frac{b^3h}{80} \qquad I_y = \frac{b^3h}{5}$$

$$\overline{I_{xy}} = \frac{b^2h^2}{120} \qquad I_{xy} = \frac{b^2h^2}{12}$$

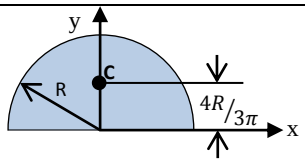
Triángulo Rectángulo



$$\bar{I}_x = \frac{bh^3}{36} \qquad \bar{I}_y = \frac{b^3h}{36} \qquad \bar{I}_{xy} = -\frac{b^2h^2}{72}$$

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{b^3h}{12} \qquad I_{xy} = \frac{b^2h^2}{24}$$

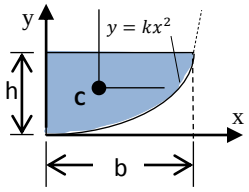
Semicírculo



$$\bar{I}_x = 0{,}1098R^4 \qquad \bar{I}_{xy} = 0$$

$$I_x = I_y = \bar{I}_y = \frac{\pi R^4}{8} \qquad I_{xy} = 0$$

Media Parábola

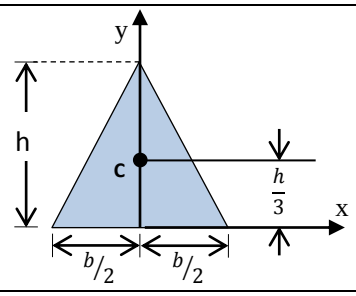


$$\bar{x} = \frac{3b}{8} \qquad \bar{y} = \frac{3h}{5} \qquad A = \frac{2}{3}bh$$

$$\bar{I}_x = \frac{8bh^3}{175} \qquad \bar{I}_y = \frac{19b^3h}{480} \qquad \bar{I}_{xy} = \frac{b^2h^2}{60}$$

$$I_x = \frac{2bh^3}{7} \qquad I_y = \frac{2b^3h}{15} \qquad I_{xy} = \frac{b^2h^2}{6}$$

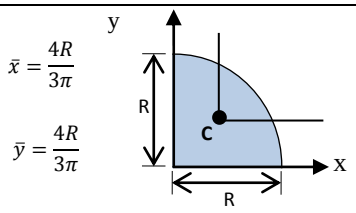
Triángulo Isósceles



$$\bar{I}_x = \frac{bh^3}{36} \qquad \bar{I}_y = \frac{b^3h}{48} \qquad \bar{I}_{xy} = 0$$

$$I_x = \frac{bh^3}{12} \qquad I_{xy} = 0$$

Cuarto de círculo

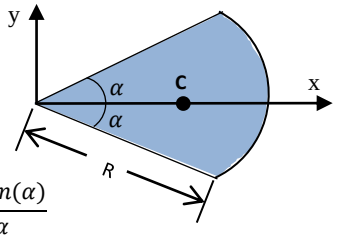


$$\bar{x} = \frac{4R}{3\pi} \qquad \bar{y} = \frac{4R}{3\pi}$$

$$\bar{I}_x = \bar{I}_y = 0{,}05488R^4 \qquad I_x = I_y = \frac{\pi R^4}{16}$$

$$\bar{I}_{xy} = -0{,}01647R^4 \qquad I_{xy} = \frac{R^4}{8}$$

Sector Circular

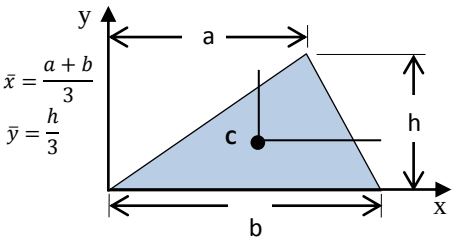


$$A = \alpha R^2 \qquad \bar{x} = \frac{2RSen(\alpha)}{3\alpha}$$

$$I_x = \bar{I}_x = \frac{R^4}{8}(2\alpha - sen2\alpha)$$

$$I_y = \frac{R^4}{8}(2\alpha + sen2\alpha) \qquad I_{xy} = 0$$

Triángulo



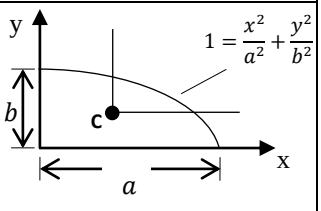
$$\bar{x} = \frac{a+b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$\bar{I}_x = \frac{bh^3}{36} \qquad I_x = \frac{bh^3}{12}$$

$$\bar{I}_y = \frac{bh}{36}(a^2 - ab + b^2) \qquad I_y = \frac{bh}{12}(a^2 + ab + b^2)$$

$$\overline{I_{xy}} = \frac{bh^2}{72}(2a - b) \qquad I_{xy} = \frac{bh^2}{24}(2a + b)$$

Cuarto de elipse



$$\bar{x} = \frac{4a}{3\pi} \qquad \bar{y} = \frac{4b}{3\pi}$$

$$\bar{I}_x = 0{,}05488ab^3 \qquad I_x = \frac{\pi ab^3}{16}$$

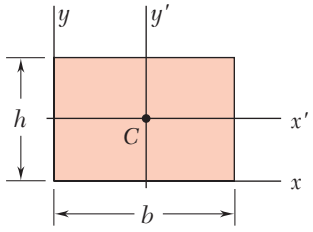
$$\bar{I}_y = 0{,}05488a^3b \qquad I_y = \frac{\pi a^3 b}{16}$$

$$\overline{I_{xy}} = -0{,}01647a^2b^2 \qquad I_{xy} = \frac{a^2b^2}{8}$$

Momentos de inercia de formas geométricas comunes

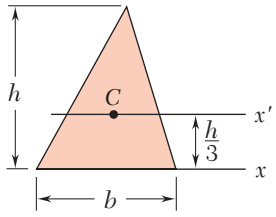
Rectángulo

$\bar{I}_{x'} = \frac{1}{12}bh^3$
$\bar{I}_{y'} = \frac{1}{12}b^3h$
$I_x = \frac{1}{3}bh^3$
$I_y = \frac{1}{3}b^3h$
$J_C = \frac{1}{12}bh(b^2 + h^2)$



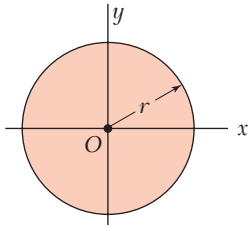
Triángulo

$\bar{I}_{x'} = \frac{1}{36}bh^3$
$I_x = \frac{1}{12}bh^3$



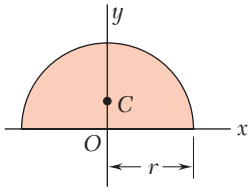
Círculo

$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$
$J_O = \frac{1}{2}\pi r^4$



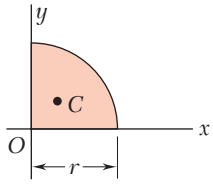
Semicírculo

$I_x = I_y = \frac{1}{8}\pi r^4$
$J_O = \frac{1}{4}\pi r^4$



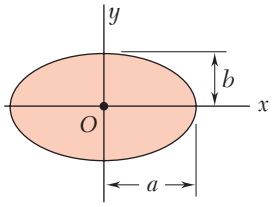
Cuarto de círculo

$I_x = I_y = \frac{1}{16}\pi r^4$
$J_O = \frac{1}{8}\pi r^4$



Elipse

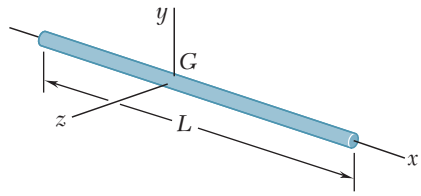
$\bar{I}_x = \frac{1}{4}\pi ab^3$
$\bar{I}_y = \frac{1}{4}\pi a^3b$
$J_O = \frac{1}{4}\pi ab(a^2 + b^2)$



Momentos de inercia de formas geométricas comunes

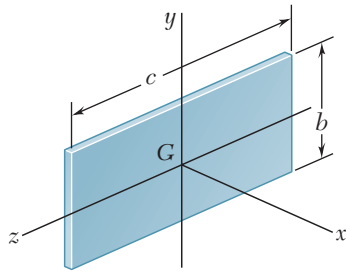
Barra delgada

$I_y = I_z = \frac{1}{12}mL^2$



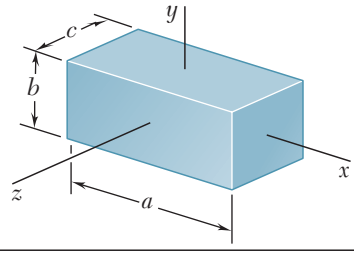
Placa rectangular delgada

$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}mc^2$
$I_z = \frac{1}{12}mb^2$



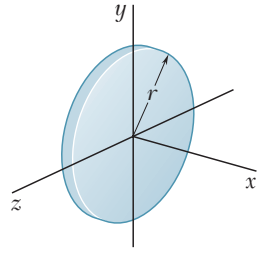
Prisma rectangular

$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}m(c^2 + a^2)$
$I_z = \frac{1}{12}m(a^2 + b^2)$



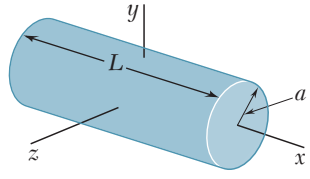
Disco delgado

$I_x = \frac{1}{2}mr^2$
$I_y = I_z = \frac{1}{4}mr^2$



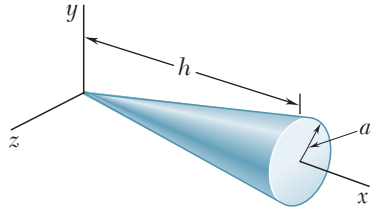
Cilindro circular

$I_x = \frac{1}{2}ma^2$
$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$



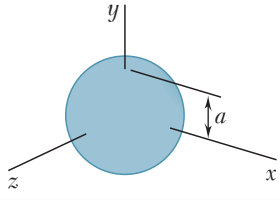
Cono circular

$I_x = \frac{3}{10}ma^2$
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$

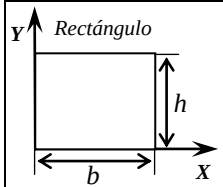
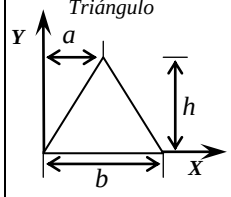
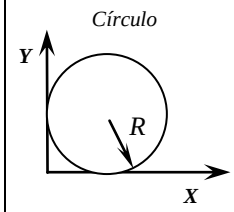
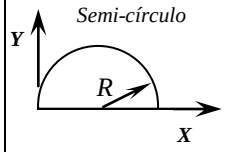
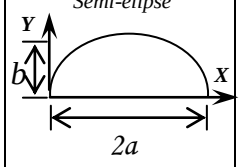
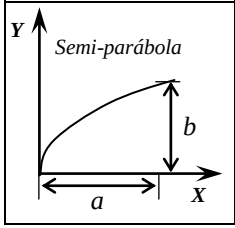


Esfera

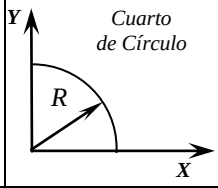
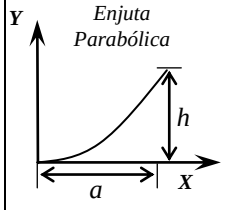
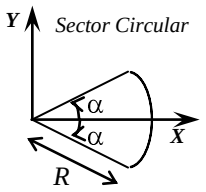
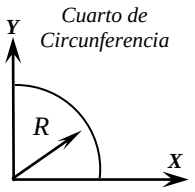
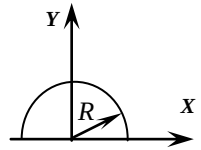
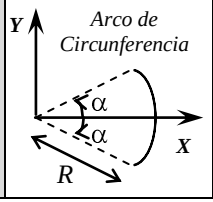
$I_x = I_y = I_z = \frac{2}{5}ma^2$



PROPIEDADES DE LAS FIGURAS PLANAS

FIGURA	ÁREA Y CENTROIDE	MOMENTO DE INERCIA	PRODUCTO DE INERCIA
	$A = bh$ $\bar{X} = \frac{b}{2}$ $\bar{Y} = \frac{h}{2}$	$I_X = \frac{bh^3}{3}$; $I_Y = \frac{b^3h}{3}$ $I_{X_C} = \frac{bh^3}{12}; I_{Y_C} = \frac{b^3h}{12}$	$I_{XY} = \frac{b^2h^2}{4}$ $I_{X_CY_C} = 0$
	$A = \frac{bh}{2}$ $\bar{X} = \frac{a+b}{3}$ $\bar{Y} = \frac{h}{3}$	$I_X = \frac{bh^3}{12}; I_{X_C} = \frac{bh^3}{36}$ $I_Y = \frac{bh}{12}(b^2 + ab + a^2)$ $I_{Y_C} = \frac{bh}{36}(b^2 - ab + a^2)$	$I_{XY} = \frac{bh^2}{24}(2a+b)$ $I_{X_CY_C} = \frac{bh^2}{72}(2a-b)$
	$A = \pi R^2$ $\bar{X} = R$ $\bar{Y} = R$	$I_X = I_Y = \frac{5\pi R^4}{4}$ $I_{X_C} = I_{Y_C} = \frac{\pi R^4}{4}$	$I_{XY} = \pi R^4$ $I_{X_CY_C} = 0$
	$A = \frac{\pi R^2}{2}$ $\bar{X} = R$ $\bar{Y} = \frac{4R}{3\pi}$	$I_X = \frac{\pi R^4}{8}; I_Y = \frac{5\pi R^4}{8}$ $I_{X_C} = \frac{R^4(9\pi^2 - 64)}{72\pi}$ $I_{Y_C} = \frac{\pi R^4}{8}$	$I_{XY} = \frac{2R^4}{3}$ $I_{X_CY_C} = 0$
	$A = \frac{\pi ab}{2}$ $\bar{X} = a$ $\bar{Y} = \frac{4b}{3\pi}$	$I_X = \frac{\pi ab^3}{8}$; $I_Y = \frac{5\pi a^3b}{8}$ $I_{X_C} = \frac{ab^3}{72\pi}(9\pi^2 - 64)$ $I_{Y_C} = \frac{\pi a^3b}{8}$	$I_{XY} = \frac{2a^2b^2}{3}$ $I_{X_CY_C} = 0$
	$A = \frac{2ab}{3}$ $\bar{X} = \frac{3a}{5}$ $\bar{Y} = \frac{3b}{8}$	$I_X = \frac{2ab^3}{15}$; $I_Y = \frac{2a^3b}{7}$ $I_{xc} = \frac{19ab^3}{480}$; $I_{yc} = \frac{8a^3b}{175}$	$I_{XY} = \frac{a^2b^2}{6}$ $I_{XCYC} = \frac{a^2b^2}{60}$

CENTROIDES DE LINEA

FIGURA	ÁREA Y CENTROIDE	MOMENTO DE INERCIA	PRODUCTO DE INERCIA
	$A = \frac{\pi R^2}{4}$ $\bar{X} = \bar{Y} = \frac{4R}{3\pi}$	$I_X = I_Y = \frac{\pi R^4}{16}$ $I_{X_C} = I_{Y_C} = \frac{R^4}{144\pi}\ 9\pi^2 - 64$	$I_{XY} = \frac{R^4}{8}$ $I_{xcyc} = \frac{R^4}{72\pi}\ 9\pi - 32$
	$A = \frac{ah}{3}$ $\bar{X} = \frac{3a}{4}$ $\bar{Y} = \frac{3h}{10}$	$I_X = \frac{ah^3}{21}$ $I_{X_C} = \frac{37ah^3}{2100}$ $I_Y = \frac{a^3h}{5}$ $I_{Y_C} = \frac{a^3h}{80}$	$I_{XY} = \frac{a^2h^2}{12}$ $I_{X_CY_C} = \frac{a^2h^2}{120}$
	$A = \alpha R^2$ $\bar{X} = \frac{2RSen\alpha}{3\alpha}$ $\bar{Y} = 0$	$I_X = I_{X_C} = \frac{R^4}{4}(\alpha - Sen\alpha Cos\alpha)$ $I_Y = \frac{R^4}{4}(\alpha + Sen\alpha Cos\alpha)$ $I_{Y_C} = \frac{R^4}{4}(\alpha + Sen\alpha Cos\alpha) - \left(\frac{2Rsen\alpha}{3\alpha}\right)^2 \cdot \alpha R^2$	$I_{XY} = 0$ $I_{X_CY_C} = 0$
		**LONGITUD** $L = \frac{\pi R}{2}$	**CENTROIDE** $\bar{X} = \bar{Y} = \frac{2R}{\pi}$
		$L = \pi R$	$\bar{X} = 0$ $\bar{Y} = \frac{2R}{\pi}$
		$L = 2\alpha R$	$\bar{X} = \frac{R\ \ Sen\alpha}{\alpha}$ $\bar{Y} = 0$